Exploiting poor randomness: RSA, DSA, and ECDSA disasters

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Textbook RSA

[Rivest Shamir Adleman 1977]

Public Key

$N = pq$ modulus

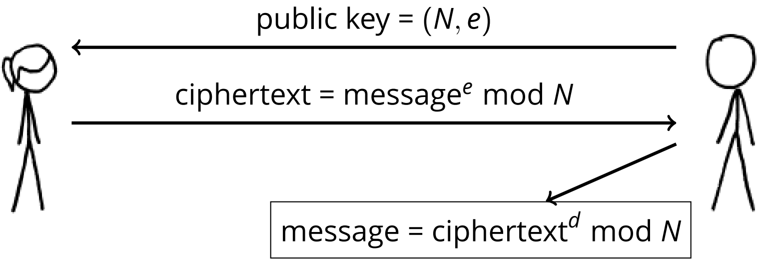
e encryption exponent

Private Key

p, q primes

d decryption exponent
$(d = e^{-1} \bmod (p-1)(q-1))$

Encryption



public key = (N, e)

ciphertext = $\text{message}^e \bmod N$

$\boxed{\text{message} = \text{ciphertext}^d \bmod N}$

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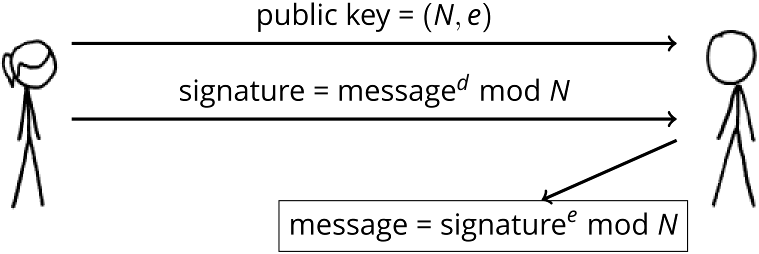
Signing



public key = (N, e)

signature = message^d mod N

$\boxed{\text{message} = \text{signature}^e \bmod N}$

Are repeated public exponents a problem?

RSA Public Keys

$N = pq$ modulus

e encryption exponent

TLS *e* values	
65537	5,689,766
17	39,637
3	19,629
35	6,272
5	418
7	201
47	94
11	80
59	77
65535	44
37	13
44611	13
13	8
65543	7
2147483647	7
65539	6
257	5

Are repeated public moduli a problem?

Public Key

$N = pq$ modulus

e encryption exponent

Private Key

p, q primes

d decryption exponent
($d = e^{-1} \bmod (p-1)(q-1)$)

- Two hosts share N: → both know private key of the other. Factorization is unique.

Hosts share the same public and private keys, and can decrypt and sign for each other.

What happens if we look for repeated moduli?

$>$ 60% of HTTPS and SSH hosts served non-unique public keys.

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Many valid (and common) reasons to share keys:

- Shared hosting situations. Virtual hosting.
- A single organization registers many domain names with the same key.
- Expired certificates that are renewed with the same key.

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Common (and unwise) reasons to share keys:

- Device default certificates/keys.
- Apparent entropy problems in key generation.

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HTTPS:
default certificates/keys:
670,000 hosts (5%)

low-entropy repeated keys:
40,000 hosts (0.3%)

SSH:
default or low-entropy keys:
1,000,000 hosts (10%)

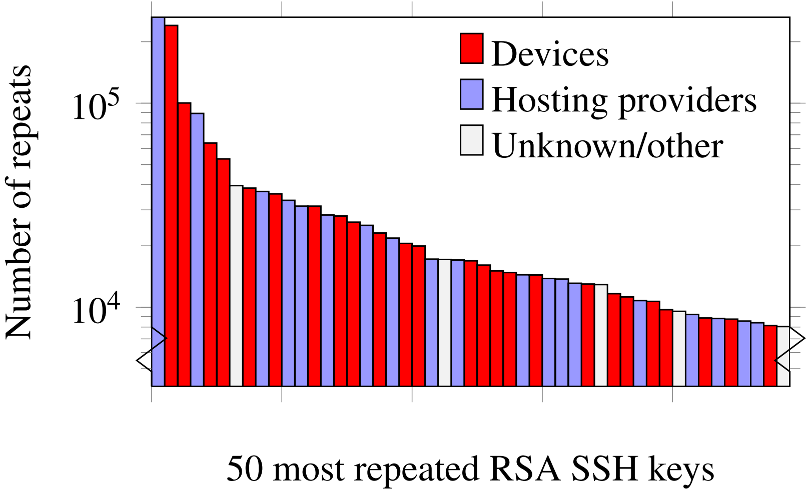
Subjects of most repeated TLS Certificates

```
C=TW, ST=HsinChu, L=HuKou, O=DrayTek Corp., OU=DrayTek Support, CN=Vigor Rou
C=UA, ST=Califonia, L=Irvine, O=Broadcom, OU=Broadband, CN=Daniel/emailAddre
C=US, ST=AL, L=Huntsville, O=ADTRAN, Inc., CN=NetVanta/emailAddress=tech.sup
C=CA, ST=Quebec, L=Gatineau, O=Axentraserver Default Certificate 863B4AB, CN
C=US, ST=California, L=Santa Clara, O=NETGEAR Inc., OU=Netgear Prosafe, CN=N
C=--, ST=SomeState, L=SomeCity, O=SomeOrganization, OU=SomeOrganizationalUni
C=US, ST=Texas, L=Round Rock, O=Dell Inc., OU=Remote Access Group, CN=iDRAC6
C=--, ST=SomeState, L=SomeCity, O=SomeOrganization, OU=SomeOrganizationalUni
C=IN, ST=WA, L=WA, O=lxlabs, OU=web, CN=*.lxlabs.com/emailAddress=sslsign@lx
C=TW, ST=none, L=Taipei, O=NetKlass Techonoloy Inc, OU=NetKlass, CN=localhos
C=--, ST=SomeState, L=SomeCity, O=SomeOrganization, OU=SomeOrganizationalUni
C=US, CN=ORname_Jungo: OpenRG Products Group
C=--, ST=SomeState, L=SomeCity, O=SomeOrganization, OU=SomeOrganizationalUni
C=LT, L=Kaunas, O=Ubiquiti Networks Inc., OU=devint, CN=ubnt/emailAddress=su
C=PL, ST=Some-State, O=Mini Webservice Ltd
C=US, ST=Texas, L=Round Rock, O=Dell Inc., OU=Remote Access Group, CN=DRAC5
C=AU, ST=Some-State, O=Internet Widgits Pty Ltd, CN=TS Series NAS
C=DE, ST=NRW, L=Wuerselen, O=LANCOM Systems, OU=Engineering, CN=www.lancom s
```

x509 Subject Alt Name of Repeated Trusted TLS Certificates

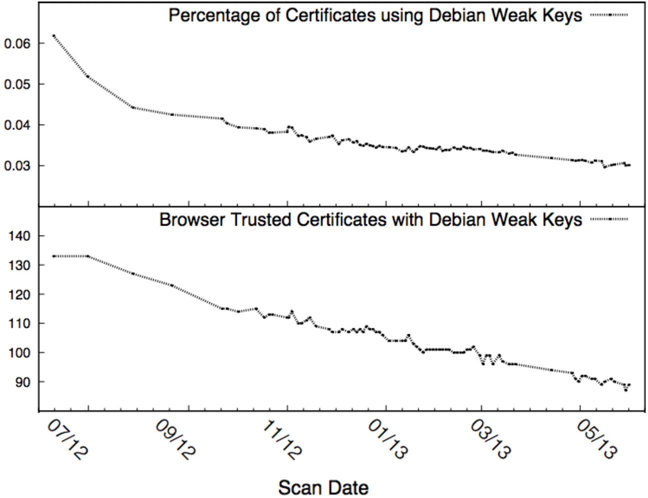
```
DNS:*.opentransfer.com, DNS:opentransfer.com
DNS:*.home.pl, DNS:home.pl
DNS:a248.e.akamai.net, DNS:*.akamaihd.net, DNS:*.akamaihd-staging.net
DNS:*.c11.hesecure.com, DNS:c11.hesecure.com
DNS:*.pair.com, DNS:pair.com
DNS:*.c12.hesecure.com, DNS:c12.hesecure.com
DNS:*.c10.hostexcellence.com, DNS:c10.hostexcellence.com
DNS:*.securesitehosting.net, DNS:securesitehosting.net
DNS:*.sslcert19.com, DNS:sslcert19.com
DNS:*.c11.ixsecure.com, DNS:c11.ixsecure.com
DNS:*.c9.hostexcellence.com, DNS:c9.hostexcellence.com
DNS:*.naviservers.net, DNS:naviservers.net
DNS:*.c10.ixwebhosting.com, DNS:c10.ixwebhosting.com
DNS:*.google.com, DNS:google.com, DNS:*.atggl.com, DNS:*.youtube.com, DNS:yo
DNS:*.hospedagem.terra.com.br
DNS:*.c8.ixwebhosting.com, DNS:c8.ixwebhosting.com
DNS:www.control.tierra.net, DNS:control.tierra.net
```

Classifying repeated SSH host keys

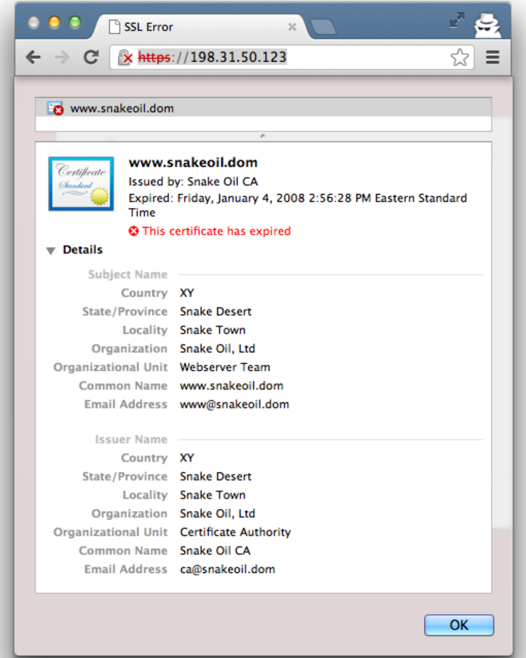


Debian OpenSSL Weak Keys

31,111 (0.34%) of RSA SSH hosts



Durumeric Wustrow Halderman 2013



www.snakeoil.dom

Issued by: Snake Oil CA

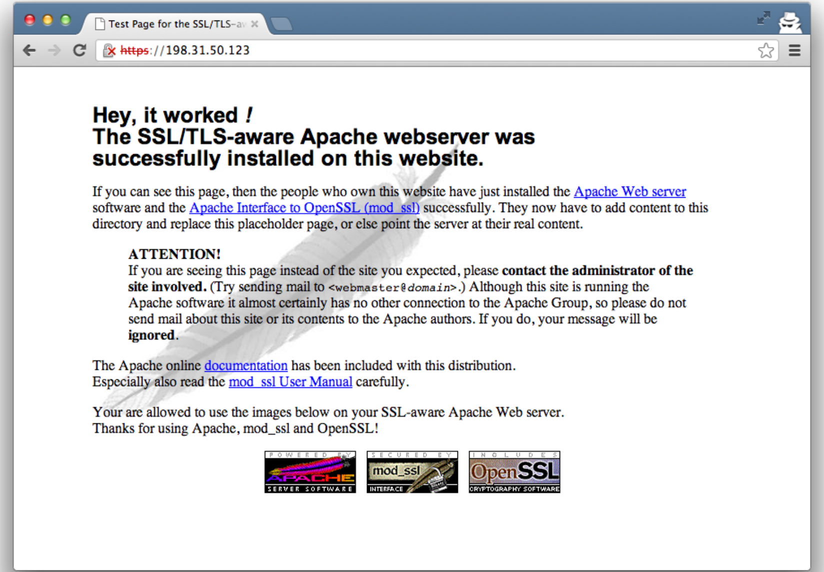
Expired: Friday, January 4, 2008 2:56:28 PM Eastern Standard Time

This certificate has expired

Details

Subject Name	
Country	XY
State/Province	Snake Desert
Locality	Snake Town
Organization	Snake Oil, Ltd
Organizational Unit	Webserver Team
Common Name	www.snakeoil.dom
Email Address	www@snakeoil.dom

Issuer Name	
Country	XY
State/Province	Snake Desert
Locality	Snake Town
Organization	Snake Oil, Ltd
Organizational Unit	Certificate Authority
Common Name	Snake Oil CA
Email Address	ca@snakeoil.dom



Hey, it worked !
The SSL/TLS-aware Apache webserver was successfully installed on this website.

If you can see this page, then the people who own this website have just installed the Apache Web server software and the Apache Interface to OpenSSL (mod_ssl) successfully. They now have to add content to this directory and replace this placeholder page, or else point the server at their real content.

> **ATTENTION!**
> If you are seeing this page instead of the site you expected, please **contact the administrator of the site involved.** (Try sending mail to `<webmaster@domain>`.) Although this site is running the Apache software it almost certainly has no other connection to the Apache Group, so please do not send mail about this site or its contents to the Apache authors. If you do, your message will be **ignored**.

The Apache online documentation has been included with this distribution.
Especially also read the mod_ssl User Manual carefully.

Your are allowed to use the images below on your SSL-aware Apache Web server.
Thanks for using Apache, mod_ssl and OpenSSL!

Two hosts share N and have different e?

Amusing Textbook RSA vulnerability.

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Looked for keys with this property, didn't find any.

What could go wrong: Shared factors

If two RSA moduli share a common factor,

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If two RSA moduli share a common factor,

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$$\gcd(N_1, N_2) = p$$

You can factor both keys with GCD algorithm.

Time to factor
768-bit RSA modulus:
2.5 calendar years
[Kleinjung et al. 2010]

Time to calculate GCD
for 1024-bit RSA moduli:
15μs

Do we actually expect to find key collisions in the wild?

Experiment: Compute GCD of each pair *M* moduli randomly chosen from *P* primes.

What *should* happen? **Nothing.**

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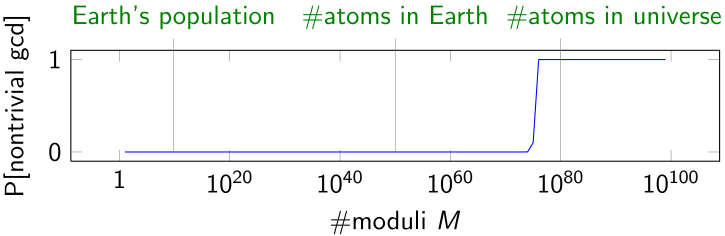
What *should* happen? **Nothing.**

Prime Number Theorem:
$\sim 10^{150}$ 512-bit primes

Birthday bound:
$\Pr[\text{nontrivial gcd}] \approx 1 - e^{-2M^2/P}$



Naively computing pairwise GCDs

Euclid's algorithm $\gcd(a, b)$

```
if $b = 0$:
        return $a$
else:
        return $\gcd(b, a \bmod b)$
```

a, b have n bits $\rightarrow$ $O(n^2)$ time.

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"Fast multiplication and its applications" Bernstein 2008

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$$\approx 1100 \text{ years}$$

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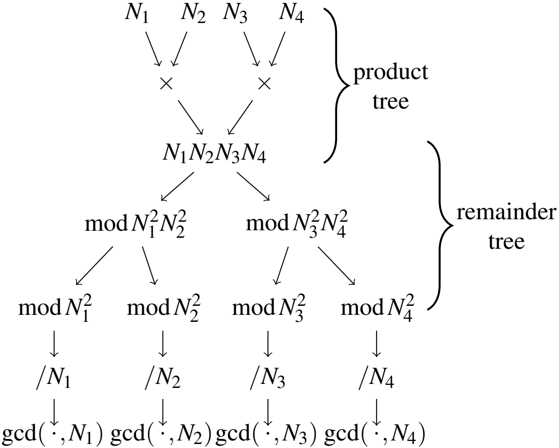
Efficiently computing pairwise GCDs

An efficient algorithm due to [Bernstein 2004].



$O(mn\,\mathrm{polylog}(mn))$ time for m n-bit integers, a few hours for datasets. Implementation available at https://factorable.net.

What happens if we compute GCDs of some RSA moduli?

What *does* happen when we GCD all the keys?

What happens if we compute GCDs of some RSA moduli?

What *does* happen when we GCD all the keys?

Compute private keys for

- 64,081 HTTPS servers (0.50%).
- 2,459 SSH servers (0.03%).
- 2 PGP users (and a few hundred invalid keys).

... only two of the factored https certificates were signed by a CA, and both are expired. The web pages aren't active.

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Subject information for certificates:

```
CN=self-signed, CN=system generated, CN=0168122008000024
CN=self-signed, CN=system generated, CN=0162092009003221
CN=self-signed, CN=system generated, CN=0162122008001051
C=CN, ST=Guangdong, O=TP-LINK Technologies CO., LTD., OU=TP-LINK SOFT, CN=TL-R478+1145D5C30089/emailAddres
C=CN, ST=Guangdong, O=TP-LINK Technologies CO., LTD., OU=TP-LINK SOFT, CN=TL-R478+139819C30089/emailAddres
CN=self-signed, CN=system generated, CN=0162072011000074
CN=self-signed, CN=system generated, CN=0162122009008149
CN=self-signed, CN=system generated, CN=0162122009000432
CN=self-signed, CN=system generated, CN=0162052010005821
CN=self-signed, CN=system generated, CN=0162072008005267
C=US, O=2Wire, OU=Gateway Device/serialNumber=360617088769, CN=Gateway Authentication
CN=self-signed, CN=system generated, CN=0162082009008123
CN=self-signed, CN=system generated, CN=0162072008005385
CN=self-signed, CN=system generated, CN=0162082008000317
C=CN, ST=Guangdong, O=TP-LINK Technologies CO., LTD., OU=TP-LINK SOFT, CN=TL-R478+3F5878C30089/emailAddres
CN=self-signed, CN=system generated, CN=0162072008005597
CN=self-signed, CN=system generated, CN=0162072010002630
CN=self-signed, CN=system generated, CN=0162032010008958
CN=109.235.129.114
CN=self-signed, CN=system generated, CN=0162072011004982
CN=217.92.30.85
CN=self-signed, CN=system generated, CN=0162112011000190
CN=self-signed, CN=system generated, CN=0162062008001934
CN=self-signed, CN=system generated, CN=0162112011004312
CN=self-signed, CN=system generated, CN=0162072011000946
C=US, ST=Oregon, L=Wilsonville, CN=141.213.19.107, O=Xerox Corporation, OU=Xerox Office Business Group,
CN=XRX0000AAD53FB7.eecs.umich.edu, CN=(141.213.19.107|XRX0000AAD53FB7.eecs.umich.edu)
CN=self-signed, CN=system generated, CN=0162102011001174
CN=self-signed, CN=system generated, CN=0168112011001015
CN=self-signed, CN=system generated, CN=0162012011000446
```

Attributing SSL and SSH vulnerabilities to implementations

Evidence strongly suggested *widespread implementation problems*.

Clue #1: Vast majority of weak keys generated by network devices:

- Juniper network security devices
- Cisco routers
- IBM server management cards
- Intel server management cards
- Innominate industrial-grade firewalls
- . . .

Identified devices from $>$ 50 manufacturers

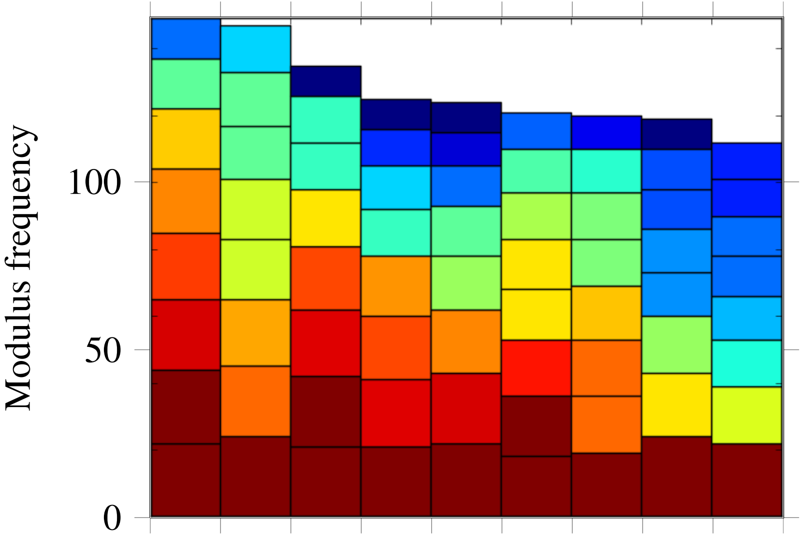
Attributing SSL and SSH vulnerabilities to implementations

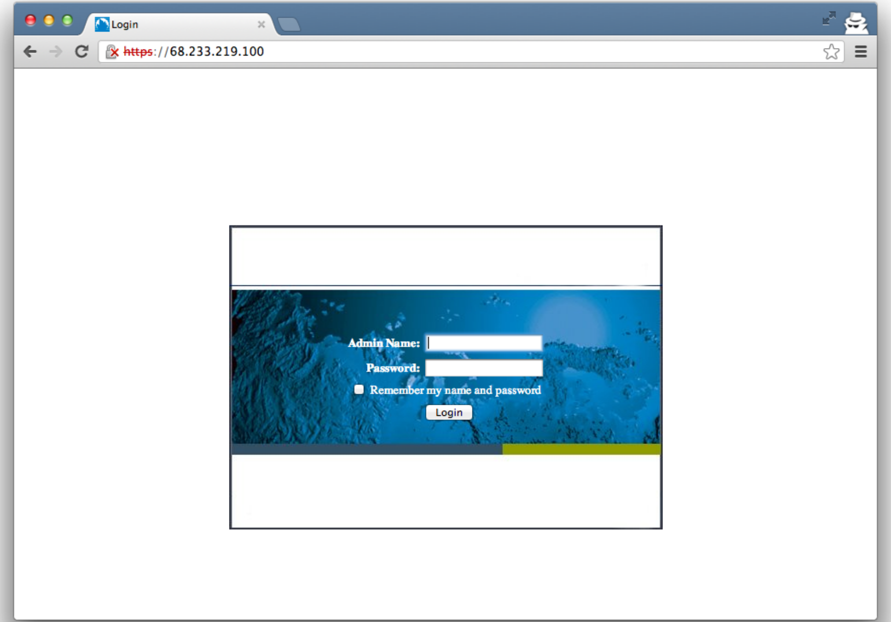
Evidence strongly suggested *widespread implementation problems*.

Clue #2: Very different behavior for different devices. Different companies, implementations, underlying software, distributions of prime factors.

Distribution of prime factors

IBM Remote Supervisor Adapter II and Bladecenter Management Module

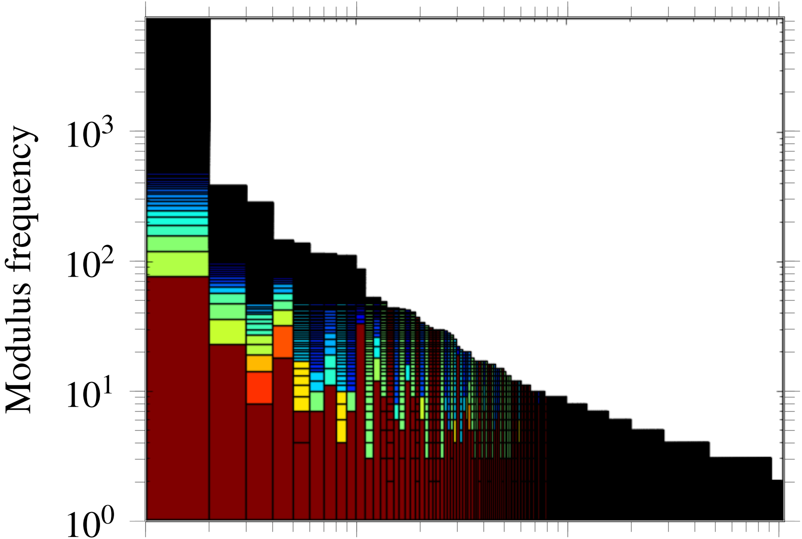




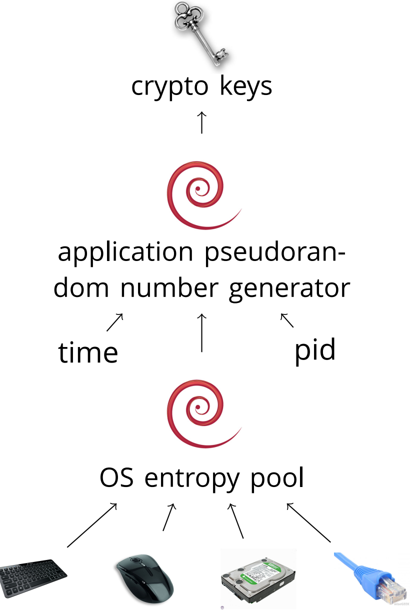
Login
https://68.233.219.100
Admin Name:
Password:
☐ Remember my name and password
Login

Distribution of prime factors

Juniper SRX branch devices



Random number generation in software



crypto keys

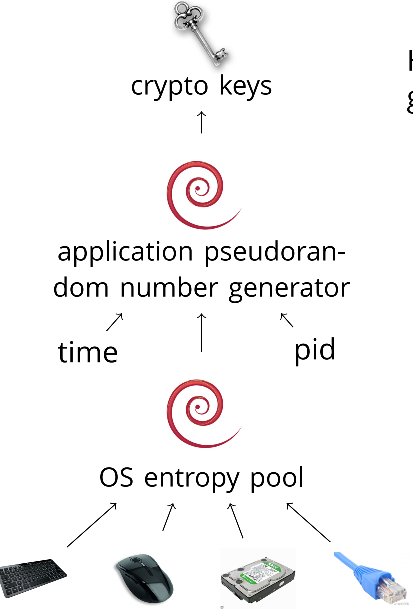
application pseudorandom number generator

time

pid

OS entropy pool

Random number generation in software



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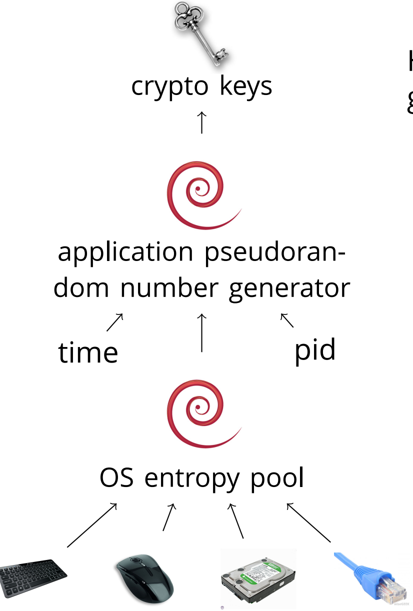
Hypothesis: Devices automatically generate crypto keys on first boot.

Random number generation in software



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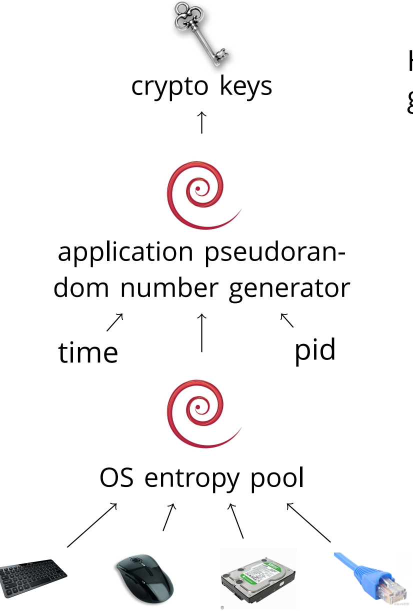
- Headless or embedded devices may lack these entropy sources.

Random number generation in software



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OS entropy pool

Hypothesis: Devices automatically generate crypto keys on first boot.

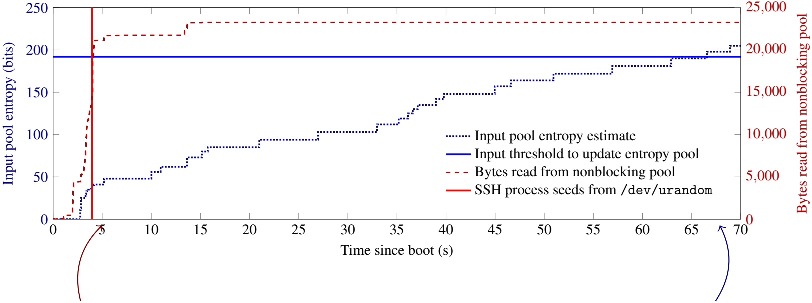
- OS random number generator may not have incorporated any entropy when queried by software.

- Headless or embedded devices may lack these entropy sources.

Linux boot-time entropy hole

Experiment: Instrument Linux kernel to track entropy estimates.

Ubuntu Server 10.04



SSH process starts

entropy pool updated

Patched since July 2012.

Generating vulnerable RSA keys in software

- Insufficiently random seeds for pseudorandom number generator $\implies$ we should see repeated keys.

```
prng.seed()
p = prng.random_prime()
q = prng.random_prime()
N = p*q
```

- We do:
  - $>$ 60% of hosts share keys
  - At least 0.3% due to bad randomness.
- Repeated keys may be a sign that implementation is vulnerable to a targeted attack.

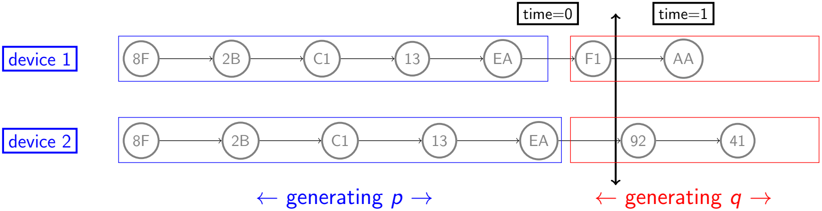
But why do we see factorable keys?

Generating factorable RSA keys in software

```
prng.seed()
p = prng.random_prime()
prng.add_randomness()
q = prng.random_prime()
N = p*q
```

OpenSSL adds time in seconds

Insufficient randomness can lead to factorable keys.

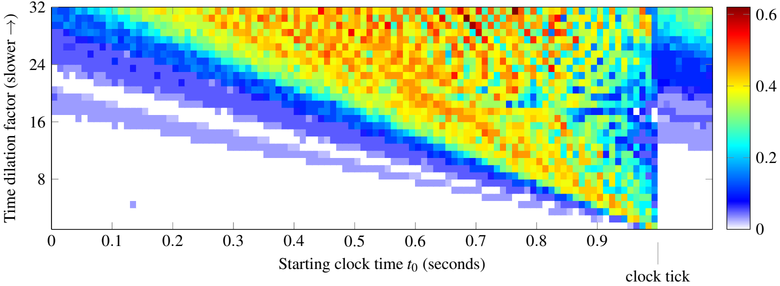


Experimentally verified OpenSSL generates factorable keys in this situation.

Experimentally generating factorable keys in OpenSSL

Experiment: Generate keys in OpenSSL with time as only entropy source.

Fraction of keys generated that we could factor



time as entropy source + asynchronous clocks → factorable keys

Unexplained oddities

Here are some prime factors of SSH keys (changed to protect the guilty):

```
d80000000000000...00000000000000000000000000000000001b3
bc0000000000000...00000000000000000000000000000000000c9
c60000000000000...00000000000000000000000000000000001ae
```

Unexplained oddities

Here are some other prime factors of HTTPS keys we found:

```
c3a64ae7fc4d4d9f75cd2a49ec2d9f7...
c3a64ae7fc4d4d9f75cd2e5f2fc56c9...
c3a64ae7fc4d4d9f75cdee869c62229...

ee93536e58a60b0f56bf95faedc7ca42a9c9809a0aae2...
ee93536e58a60b0f56bf95faedc7ca42a9c9809a2cf5b...
ee93536e58a60b0f56bf95faedc7ca42a9c9809aad4a8...
ee93536e58a60b0f56bf95faedc7ca42a9c9809abb02d...
ee93536e58a60b0f56bf95faedc7ca42a9c9809acef6f...
```

PGP

Why did GCD factor two PGP keys?

They were both $>$ 10 years old.

PGP uses `/dev/random` to generate keys.

Textbook Diffie-Hellman

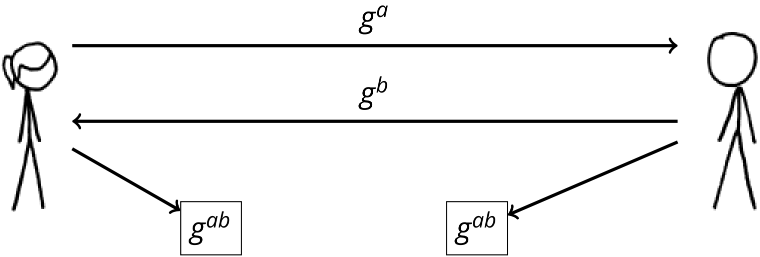
[Diffie Hellman 1976]

Public Parameters

G a group (e.g. $\mathbb{F}_p$, or an elliptic curve)

g group generator

Key Exchange



Is a repeated g^a a vulnerability?

- Yes, if unrelated parties know discrete log/private key a.
- Yes, if repeated values signal entropy issues.

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5.4*M* key exchanges 5.2*M* unique values.

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Possible explanation: OpenSSL using ephemeral-static ECDH. (Keys ephemeral per application instance and not per handshake.)

121,000 values presented by $>$ 1 IP address, most common on 2,000 hosts.

Mostly shared hosting. One Netasq device always presents same key for ECDHE.

The DSA Algorithm

DSA Public Key

p prime

q prime, divides $(p-1)$

g generator of subgroup of order q mod p

y = g^x mod p

Private Key

x private key

Verify

$$u_1 = H(m)s^{-1} \bmod q$$
$$u_2 = rs^{-1} \bmod q$$
$$r \overset{?}{=} g^{u_1}y^{u_2} \bmod p \bmod q$$

Sign

Generate random k.

$$r = g^k \bmod p \bmod q$$
$$s = k^{-1}(H(m) + xr) \bmod q$$

ECDSA

ECDSA Public Key

G generator $\in E(\mathbb{F}_p)$

$Q = dG$

Private Key

d private key

Sign

Generate random k.

$(x, y) = kG$ $r = x \bmod n$

$s = k^{-1}(H(m) + dr) \bmod n$

What could go wrong: Repeated keys

DSA public keys

Public key

p prime

q prime, divides $(p-1)$

g generator of subgroup of order q mod p

$y = g^x \bmod p$

- Two hosts have same public key → both know private key of the other.

What could go wrong: Weak DSA signature nonce

Public Key

p, q, g domain parameters

$y = g^x \bmod p$

Private Key

x private key

Signature: (r, s_1)

$r = g^{k} \bmod p \bmod q$

$s_1 = k^{-1}(H(m_1) + xr) \bmod q$

- DSA nonce known → easily compute private key.

What could go wrong: Weak DSA signature nonce

Public Key

p, q, g domain parameters

$y = g^x \bmod p$

Private Key

x private key

Signature: (r, s_1)

$r = g^k \bmod p \bmod q$

$s_1 = k^{-1}(H(m_1) + xr) \bmod q$

Signature: (r, s_2)

$r = g^k \bmod p \bmod q$

$s_2 = k^{-1}(H(m_2) + xr) \bmod q$

- DSA nonce known → easily compute private key.

$$s_1 - s_2 = k^{-1}(H(m_1) - H(m_2)) \bmod q$$

- DSA nonce reused to sign distinct messages → easily compute nonce.

What happens if we look for repeated DSA nonces?

Compute private keys for

- 105,728 (1.03%) of SSH DSA servers.
- 133 Bitcoin addresses.

Generating weak DSA signatures

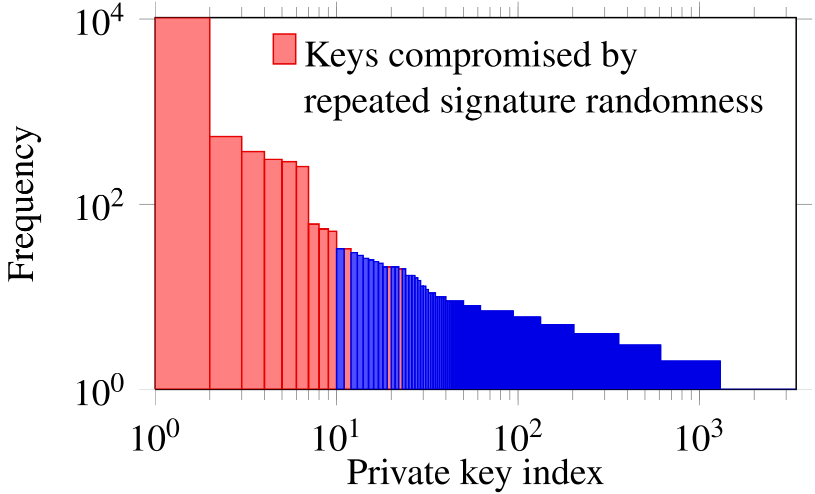
Step 1: Low-entropy DSA key generation

Step 2: Low-entropy seed for PRNG generating signature nonce.

Host 1	Host 2
50	84
58	24
9	13
36	89
84	85
24	68
13	52
89	69
85	47

Step 3: Two sequences in same state → colliding nonces.

Compromised DSA keys from Gigaset DSL routers

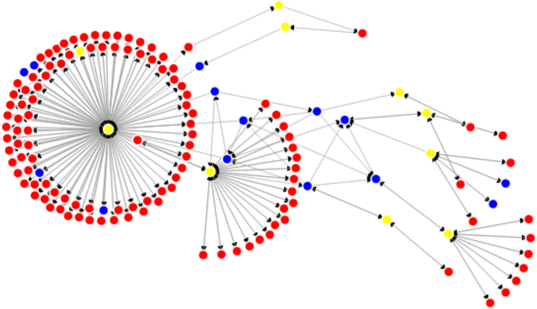


Bitcoin

Several explanations so far:

- Android Java RNG vulnerability publicized August 2013.
- Test implementations.
- Developer error in uncommon bitcoin implementations.

Bitcoin address `1HKywxiL4JziqXrzLKhmB6a74ma6kxbSDj` has stolen 59 bitcoins from weak addresses so far.



red = vulnerable keys

Disclosure for HTTPS and SSH vulnerabilities

- Wrote disclosures to 61 companies.
- 13 had Security Incident Response Team contact information available.
- Received responses from 28.
- 13 told us they fixed the problem
- 5 informed us of security advisories
- Coordinated through US-CERT, ICS CERT, JP-CERT
- Linux kernel has been patched.

Vendor responses

“When running the testing, would you be able to provide the software on the .. and the firmware on the ... along with model numbers on the ....”

“Attached is a document on the security the ... uses.” (It was empty.)

“Would you be able to provide the login credentials for the 3 test IP Addresses you provided. I would like to login to the device to gather the software and firmware installed.”

“Hi. What is your billing address, so that I can fwd your email to the appropriate Account Executive.”

“some IT auditor somewhere is handed your paper, alarm bells sound in his or her head, and things start to get unnecessarily emergent, network admins start calling us, CSIRTs start engaging us to figure out what’s going one, etc., etc.”

Disclosure to end-users

- Attempted to contact end-users with signed certificates sharing keys with default certificates.

- Certificates belonged to Fortune 500 companies, insurance providers, law firms, a major public transit authority, and the US Navy.

Media Coverage

Lenstra, Hughes, Augier, Bos, Kleinjung, Wachter

Flaw Found in an Online Encryption Method

By JOHN MARKOFF

Published: February 14, 2012

SAN FRANCISCO — A team of European and American mathematicians and cryptographers have discovered an unexpected weakness in the encryption system widely used worldwide for online shopping, banking, e-mail and other Internet services intended to remain private and secure.

The flaw — which involves a small but measurable number of cases — has to do with the way the system generates random numbers, which are used to make it practically impossible for an attacker to unscramble digital messages. While it can affect the transactions of individual Internet users, there is nothing an individual can do about it. The operators of large Web sites will need to make changes to ensure the security of their systems, the researchers said.

The potential danger of the flaw is that even though the number of users affected by the flaw may be small, confidence in the security of Web transactions is reduced, the authors said.

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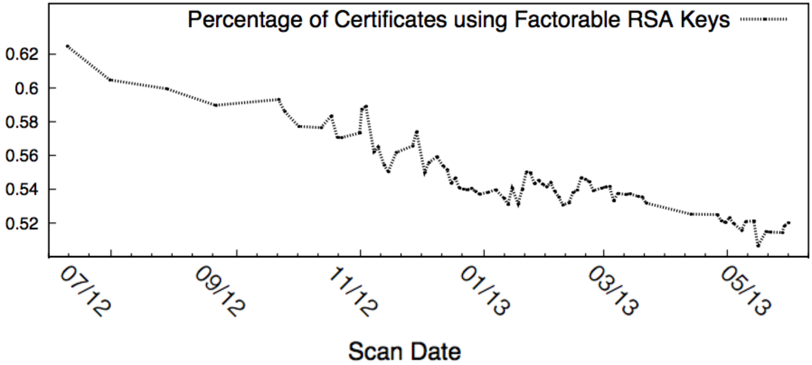
Factorable TLS keys over time



Mining your Ps and Qs: Widespread Weak Keys in Network Devices Nadia Heninger, Zakir Durumeric, Eric Wustrow, and J. Alex Halderman *Usenix Security 2012* https://factorable.net

"Ron was wrong, Whit is right" published as *Public Keys* Arjen K. Lenstra, James P. Hughes, Maxime Augier, Joppe W. Bos, Thorsten Kleinjung, and Christophe Wachter *Crypto 2012*

Elliptic Curve Cryptography in Practice Joppe W. Bos, J. Alex Halderman, Nadia Heninger, Jonathan Moore, Michael Naehrig, and Eric Wustrow *Financial Cryptography 2014*